y is done first followed by the principal part of the integral with respect to x, then $P' = (I_{max}B_0lV)/2$. This is the induction drag multiplied by the velocity as was shown to be generally true in Eq. (8). It should be noted that different results are obtained from those above if one integrates first with respect to x (taking the principal value) and then follows with integration over y. This is not unusual because in applying the principal value of an integral to a physical problem, it is done because the result does have a physical meaning.

The two terms in Eq. (1) involving $h_x{}^2$ and $h_y{}^2$ are generally several orders of magnitude smaller for typical satellite conditions than the induction drag. Since the work in Ref. 1 was done entirely from a numerical viewpoint (numerical evaluation of a divergent integral), and apparently the three terms were not considered individually, it is therefore apparent why Chu and Gross concluded that wave drag is part of the induction drag. However, the analytical investigation presented here makes it clear that their conclusion was not correct.

References

[1] Chu, C. K. and Gross, R. A., "Alfvén Waves and Induction Drag on Long Cylindrical Satellites," *AIAA Journal*, Vol. 4, No. 12, Dec. 1966, pp. 2209–2214.

[2] Drell, S. D., Foley, H. M., and Ruderman, M. A., "Drag and Propulsion of Large Satellites in the Ionosphere: An Alfvén Propulsion Engine in Space," *Journal of Geophysical Research*, Vol. 70, 1965, pp. 3131–3145.

Connection between Lift and Particle Displacement

Kenji Inouye*
National Aerospace Laboratory, Jindaiji-Machi, Chofu, Tokyo, Japan

Introduction

IN the flow around a lifting body, the fluid particles which pass the low-pressure side go more quickly than those which pass the high-pressure side. If we assume the flow is uniform at infinity, the difference between the x-coordinates of two particles which start simultaneously far upstream, one passing far above the body, the other far below, results in a finite value at infinity downstream. This displacement has some correlation with the lift exerted on the body. Recently, Corrsin has made a conjecture on the connection between the lift and the particle displacement.[1] His analytical result is in good agreement with his numerical calculation. His analysis, however, has been based on an assumption that the changes of y-coordinates of the fluid particles can be neglected. This is not true in large scale of time. Therefore, there has been left some ambiguity in the validity of his result far downstream.

In the present paper, it is shown that his conjecture can be proved without the assumption stated previously, by use of the stream function as an integral of the equations of motion of the fluid particle. Similar formulation can be found in the

* Chief, Aerothermodynamics Section, First Aerodynamics Division.

analyses of drift in the flow with no circulation around a circular cylinder by Darwin and around a sphere by Lighthill.[2,3] In the following analysis, the prime is used in order to denote the dimensional quantities which shall be nondimensionalized later.

Formulation of the Problem

We consider a cylindrical body with the circulation Γ in a uniform flow with the velocity U. If we restrict our consideration to the region sufficiently far from the body, we need consider only the lifting vortex filament located at the origin, because the effect of the shape of the body decreases with the distance from the body r' as r'^{-2}.

The flowfield is described in terms of the stream function ψ' as

$$\psi' = Ur' \sin\theta + (\Gamma/2\pi) \ln r' \tag{1}$$

$$dr'/dt' = (1/r')(\partial\psi'/\partial\theta) = U\cos\theta \tag{2}$$

$$\frac{d\theta}{dt'} = -\frac{1}{r'}\frac{\partial\psi'}{\partial r'} = -\frac{1}{r'}\left(U\sin\theta + \frac{\Gamma}{2\pi}\frac{1}{r'}\right) \tag{3}$$

where (r',θ) is the polar coordinate with the origin at the vortex, t' the time, dr'/dt' and $d\theta/dt'$ the velocity components of the fluid particle in the Lagrangian sense. It is convenient to use the dimensionless variables r, t, and ψ, defined by

$$r = (2\pi U/\Gamma)r' \tag{4a}$$

$$t = (2\pi U^2/\Gamma)t' \tag{4b}$$

$$\psi = (2\pi/\Gamma)\psi' \tag{4c}$$

Then, Eqs. (1–3) become

$$\psi = r\sin\theta + \ln r \tag{5}$$

$$dr/dt = \cos\theta \tag{6}$$

$$d\theta/dt = -(1/r)(\sin\theta + 1/r) \tag{7}$$

The Eqs. (6) and (7) constitute a set of equations which describe the motion of the fluid particle, and the stream function is one of its integrals

$$\psi = r\sin\theta + \ln r = \alpha \tag{8}$$

where the parameter α is the constant of motion and its value can be regarded as a label for each streamline.

Eliminating θ from Eqs. (7) and (8), we obtain

$$dr/dt = \pm[1 - (1/r^2)(\alpha - \ln r)^2]^{1/2} \tag{9}$$

Integrating this equation, we get the solution

$$t = \pm \int \frac{r\,dr}{[r^2 - (\alpha - \ln r)^2]^{1/2}} \tag{10}$$

This is the drift function defined by Lighthill.[3]

Here we define a Cartesian coordinate (x,y) by

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta \tag{11}$$

The quantity we are interested in is the displacement of the particle in the x direction, referred to axes in which the infinite parts of the fluids are at rest. For this, what is required is $X = x - t$. This quantity has been named the drift by Darwin.[2] Combining Eqs. (6) and (9) with (11), we get

$$x = [r^2 - (\alpha - \ln r)^2]^{1/2} \tag{12}$$

From Eqs. (10) and (11) we have

$$X = [r^2 - (\alpha - \ln r)^2]^{1/2} - \int_{r_1}^{r} \frac{r\,dr}{[r^2 - (\alpha - \ln r)^2]^{1/2}} =$$
$$\int_{r_1}^{r} \frac{(\alpha - \ln r)\,dr}{r[r^2 - (\alpha - \ln r)^2]^{1/2}} \tag{13}$$

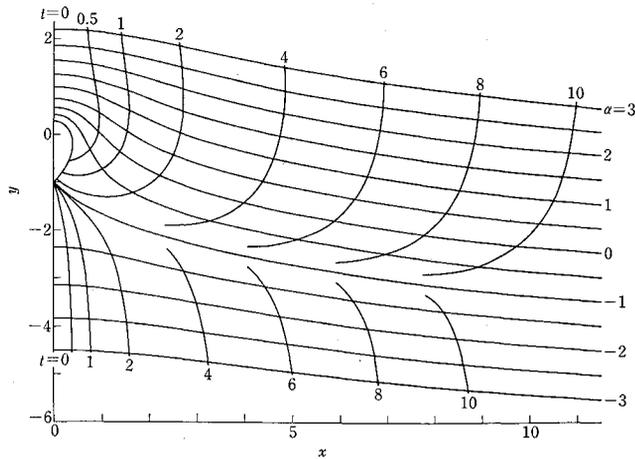where we are considering the fluid particles which start from

Fig. 1 The irrotational flow past a vortex : streamlines and surfaces t = constant. The latter are the shapes into which planes of fluid initially coincide with the plane $x = 0$ would be distorted as they passed over the vortex. Numerical results.

the point on the y-axis at the time $t = 0$ and r_1 is the largest real root of the equations $r + \ln r = \alpha$ (for $\alpha > -1$) or $-r + \ln r = \alpha$ (for $\alpha < -1$). The integral in Eq. (13), however, cannot be carried out easily. We make, therefore, an approximation. The results of the numerical calculation for the positions of fluid particles on streamlines are shown in Fig. 1 as an illustration. These results are obtained by integrating Eqs. (6) and (7) numerically.

Asymptotic Solution

The constant of motion α, which has not been used in Corrsin's analysis, represents the remoteness of the path of the particle from the origin. Because we are considering the flowfield far from the origin, we can assume the values of $|\alpha|$ are very large.

For $|\alpha| \gg 1$, $r \geq r_1 = |\alpha| + O(\ln |\alpha|)$ and therefore $r \gg \ln r$ and we can neglect the terms of $\ln r$ comparing to r in Eq. (13). Then, Eq. (13) can be approximated by

$$X = \int_{|\alpha|}^{r} \frac{(\alpha - \ln r)dr}{r(r^2 - \alpha^2)^{1/2}} \tag{14}$$

Integrating Eq. (14) by parts, we obtain

$$X = 1/|\alpha| \{(\pi/2)[\alpha - \ln(|\alpha|/2)] - (\alpha - \ln r) \operatorname{arccosec} (r/|\alpha|) - I\} \tag{15}$$
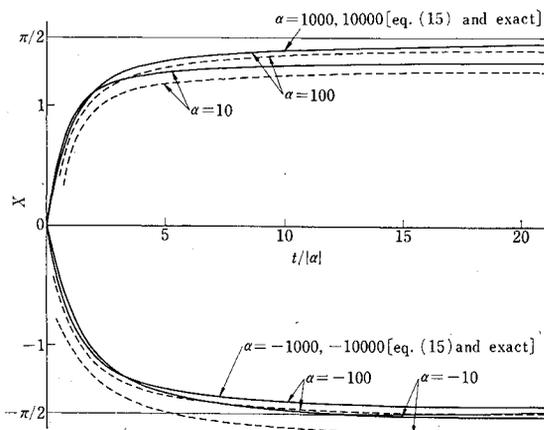

Fig. 2 Variation of the drift X for the flow past a vortex, along different streamlines, which are identified on the figure by the values of α noted on each curve. ———, exact numerical results; ------, approximate solution Eq. 15.

where

$$I = \int_0^{|\alpha|/r} \frac{\arcsin \xi}{\xi} d\xi =$$
$$(|\alpha|/r) + (1/2.3^2)(|\alpha|/r)^3 + (1.3/2.4.5^2)(|\alpha|/r)^5 + \ldots$$

The total drift $X(\alpha)$ is obtained as

$$X(\alpha) = \lim_{r \to \infty} X = (\pi/2|\alpha|)[\alpha - \ln(|\alpha|/2)]$$

Taking into account the upstream-downstream symmetry, we get the distance for the x separation of two fluid particles which start with zero x separation far upstream, one passing far above the body, the other far below ($\alpha = \pm|\alpha|$, $|\alpha| \gg 1$)

$$x_U - x_L = 2(X_U - X_L) = 2\pi$$

This expression is independent of α. Inverting this into the dimensional form by the definition (4), we have

$$x_U' - x_L' = \Gamma/U \tag{16}$$

This is just the conjecture given by Corrsin.[1] By the use of the Kutta-Joukowsky law, Eq. (16) relates particle displacement to lift L per unit span by

$$x_U' - x_L' = L/\rho U^2$$

where ρ is the density of the fluid. In Fig. 2, the approximate solution (15) is compared with the exact numerical solutions for several values of α. The exact results are obtained by integrating Eqs. (6) and (7) numerically. The agreement is good for large values of $|\alpha|$.

References

[1] Corrsin, S., "Conjecture on a Connection between Lift and Particle Displacement," *AIAA Journal*, Vol. 6, No. 9, Sept. 1968, pp. 1811–1812.

[2] Darwin, C. G., "Notes on Hydrodynamics," *Proceedings of the Cambridge Philosophical Society*, Vol. 49, Part 2, April 1953, pp. 342–354.

[3] Lighthill, M. J., "Drift," *Journal of Fluid Mechanics*, Vol. 1, Part 1, May 1956, pp. 31–53; also corrigenda, *Journal of Fluid Mechanics*, Vol. 2, Part 3, May 1957, pp. 311–312.

Direct Measurement of the Thermal Conductivity of Shock Heated Argon

R. EWALD* AND H. GRÖNIG†

Institut für Allgemeine Mechanik, Technische Hochschule, Aachen, Germany

DESPITE a considerable amount of theoretical and experimental work on thermal conductivity of gases at elevated temperatures there is still a lack of its direct measurement. Almost all authors have assumed a thermal conductivity-temperature relationship according to $\lambda \sim T^\sigma$ (Refs. 1–5). This relationship then served to determine σ experimentally. The general procedure applied by all these authors has been to calculate the thermal boundary-layer equations using

$$\lambda = \lambda_0(T/T_0)^\sigma$$

where λ_0 represents the known thermal conductivity at a reference temperature T_0. They then vary σ until the mea-